An asset is purchased for 40,000/-. it has an expected life of 4 years & no salvage value at the end of life. The purchaser intends to recover the 40,000 invested over 4 year + the interest. The

10%

Eq. Annual payment $A = \overset{40k}{P}(A/P, 10, 4)$

$= \underline{12619/-}$

$A = \dfrac{P \times i \times (1+i)^n}{(1+i)^n - 1}$

End of Period	Capital not recovered by end of Period	Interest Due	Amount of Capital recovered	Period Capital recovery charges.
0	40k	—	—	—
1	31,381.17	4k	8618.83	12618.83
2	21900.45	3138.12	9480.72	12618.83
3	11471.67	2190.05	10428.79	—"—
4	0	1147.17	11471.67	12618.83
		10,475.33	40k	50475.33

50% ↑ in this

0	40k	—	—	—
1	25071.75	4k	14928.25	18928.25
2	~~25~~ 8650.68	2507.18	16421.07	18928.25
3	0	865.07	8650.68	9515.74

80k, 48 equal quarterly (for every 3 months). $3\% = i$ ②

$A = 80k\,(A/P, 3, 48) = 3166.22$

interest value $= 3\%$ of $80k = 2400$.

Capital Recovery Calculations.

Equal Annual Cost $= P(A/P, i, N) - S(A/F, i, N)$ ↓①

$$A/P, i, N = \frac{i(1+i)^n}{(1+i)^n - 1} \quad ②$$

$$A/F, i, n = \frac{i}{(1+i)^n - 1} \quad ③$$

② $-i$

$$\frac{i(1+i)^n}{(1+i)^n - 1} - i = \frac{i(1+i)^n - i(1+i)^n + i}{(1+i)^n - 1}$$

$$= \frac{i}{(1+i)^n - 1}$$

$\therefore \ [A/F, i, n] = [A/P, i, n] - i$

$\therefore \ EAC = P(A/P, i, n) - S[(A/P, i, n) - i]$

$$\boxed{EAC = (A/P, i, n)(P - S) + Si}$$

③

Consider an asset that costs 60k & has a 20k salvage value. The net amount that must be recovered from annuity payments is P - S = 60k - 20k = 40k. The remaining portion of the purchase price is returned by receipt of the salvage value 20k. However the purchaser is deprived of the use of the 20k during the life of the asset, so interest is owed on this amount bcz it represents unrecovered capital. The SI term in ⑤ accounts for this interest payment.

$$EAC = (60k - 20k)(A/P, 10, 4) + 20k(0.10)$$

$$= 40k(0.31547) + 2k$$

$$= 12619 + 2k = 14619.$$

Also,

$$EAC = P(A/P, 10, 4) - S(A/F, 10, 4)$$

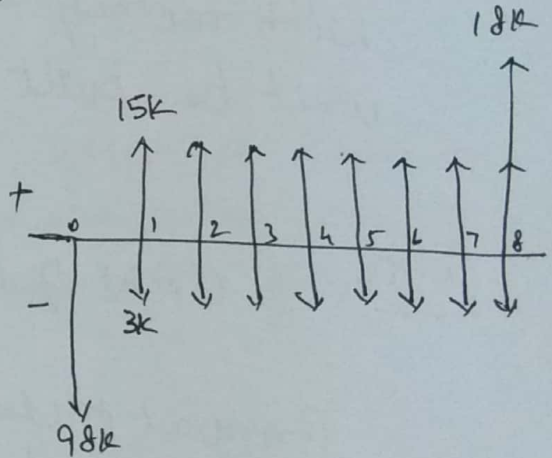
$$= 60k(0.31547) - 20k(0.21547)$$

$$= 18928 - 4309 = 14619.$$

Net Annual Worth of a single project.

⇒ The purchase of a truck with an operator's platform on a telescoping hydraulic boom will reduce labour costs for sign installations by 15k/year. The price of the boom truck is 93,000 & its operating costs will exceed those of present equipment by 250/month. The salvage value is expected to be 18000 in 8 years. Should the boom truck be purchased when the current available interest rate is 7%.

Soln:



$$EAW = -93k\,(A/P, 7, 8) + 18k\,(A/F, 7, 8) - 3k + 15k$$

$$= -1820$$

$$EAW = \text{Annual savings} - \text{Capital recovery costs}$$

$$= 15k - 3k - \{[(P-S)(A/P,7,8)] + Si\}$$

$$= 12k - \{[(93k - 18k)(A/P,7,8)] + (18k \times 0.07)\}$$

$$= -1820$$

A supplier of laboratory equipment estimates that profit from sales should increase by 20k/year if a mobile demonstration unit is built. A large unit with sleeping accommodations for the driver will cost 97k while a smaller unit without sleeping quarters will be 63k. Salvage values for the large & small unit after 5 years of use will be 9700 & 3.5k respectively. Loading costs saved by the larger unit should amount to 11k annually. But its yearly transportation cost will exceed those of the smaller unit by 3.1k. With money at 9%, should a mobile demonstration unit be built? Also if so, which size is preferable.

Soln: EAW of the large mobile demo unit.

Annual increase in profit	20k
Savings in loading costs over smaller unit / year	11k
Extra transportation costs over smaller unit / year	-3.1k
Capital recovery	- 23.317k

5

EAW of small demo unit.

Annual increase in profit	20k
Capital recovery cost	−15612
[(P−S)(A/P, 9, 5)] + (3500×0.07)	
Net AW	= 4388

Asset life:

Ownership life (or) Service life.

It is the period of time an asset is kept in service by the owner(s). Implied is a period useful service from the time of purchase until disposal.

"Accounting life" is a life expectancy based primarily on bookkeeping & ~~tax~~ tax considerations. It may or may not correspond to the period of usefulness & economic desirability.

"Economic life" is the time period that minimizes the assets total equivalent annual cost (or) max's its equivalent net annual income. At the end of this period. The asset would be displaced by a more profitable replacement if service were still required. "optimal replacement interval"

" Alternative with equal annual costs"

A machine needed for 3years can be purchased for 77662 & sold at the end of the period for about 25000. A comparable machine can be leased for 30K/year. If a firm expects a return of 20% on investments. should it buy or lease the machine when end of year payments are expected.

Soln:

EAC to buy. $= [(77662 - 25000)(A/P, 20, 3)] + (25000 \times 0.2)$

$= 30,000/-$

Cost of lease is 30,000/- [Given]

$\therefore$ $\boxed{\text{Cost of buy} = \text{Cost of lease}}$

comparison of Assets with unequal lives.

Two models of small machines perform the same function. Machine A has a low initial costs of 9500, relatively high operating costs of 1900/year more than those of the Machine B. & a short life of 4 years. The more expensive Machine B costs 25100 & can be kept in service economically for 8 years. The scrap value from either machine at the end of its life will barely covers its removal cost. Which is preffered when the minimum attractive rate of return is 8%.

Soln: Machine A

$$EAC = 1900 + 9500(A/P, 8, 4)$$

$$= 4768$$

Machine B

$$EAC = 25100(A/P, 8, 8) = 4368.$$

Machine B has a lower annual cost for service during the next 8 years & it is preffered

Use of a sinking fund.

Accumulation is called sinking fund.

Equivalent uniform payments when interest rate vary.

A Deposit of 5000 was made 4 years ago & a withdrawal of 2000 was made 2 years ago. The prevailing interest rate for first year was 6%. & the rate was increased by 1%.

Soln:

End of year	Deposit (or) withdrawal	i during year i%	Balance in account at end of year
0	5000	–	–
1		6	5000(F/P, 6, 1) = 5300
2	– 2000	7	5300(F/P, 7, 1) – 2k = 3671
3		8	371(F/P, 8, 1) = 3965
4		9	3965(F/P, 9, 1) = 4321.3

(2) (7)

An asset was purchased 5 year ago for 52K. It was expected to have an economic life of 8 years at which time its salvage value would be 4000. If the function that the asset was serving is no longer needed, for what price must it be sold now to recover the invested capital when $i = 12\%$.

Soln:

$$EAC = (P-S)(A/P, 12, 8) + S(0.12) = 10142.40/-$$

Unrecovered capital at the end of 5^{th} year is the PW of the last 3 years of the EAC annuity + Discounted Slavage value

$$\text{Unrecovered capital (5 years)} = 10142.40\,(P/A, 12, 3) + 4000\,(P/F, 12, 3)$$

$$= 24361 + 2847 = 27208$$

Alternate Soln:

Selling price (8 years − 5]

$$= 48000(A/P, 12, 8)(P/A, 12, 3) + 4K$$

$$= 23208 + 4K = 27208.$$

A city maintenance crew has had experience with a conventional backhoe that suggests that its service life is 6 years. A newly designed machine costs 50% more than the conventional machine but is quieter in operation, which will make it more adaptable to residential neighbourhoods. Both machines will have about the same operating costs & salvage costs are expected to a negligible. What will the service life of the new backhoe have to be to make its cost comparable to that of the conventional machine at $i = 10\%$.

<u>Soln:</u> Since the machines will apparently have different lives, it is logical to attack the problem by using an annual worth evaluation, assuming that replacement costs will be comparable to current costs.

i) Equating two m/c's annual worths,

$$AW_{con} = AW_{new} = P_{con}(A/P, 10, 6) = P_{new}(A/P, 10, n)$$

But W.K.T $\quad P_{new} = 1.5 P_{con}$.

$$\therefore P_{con}(A/P, 10, 6) = 1.5 P_{con}(A/P, 10, n)$$

$$\therefore \frac{1}{1.5}(A/P, 10, 6) = (A/P, 10, n)$$

0.1532

$$0.667(0.22961) = (A/P, 10, N)$$

Interpolating in the tables, we find that N lies b/w 11 & 12.

$$\therefore N = 11 + \frac{0.15396 - 0.1532}{0.15396 - 0.14676} = 11.1 \text{ years}.$$

⑧ *

A short concrete canal can be constructed as part of flood control project. the placement of a large galvanized culvert will serve the same function. The cost of the canal. Which will last indefinitely is 7500/- & maintenance cost will average 400/year. A culvert which will have to be replaced every 30 year will cost 40k & have an annual maintenance cost of 700. Salvage values are negligible for both alternatives & the govt interest rate is 6%. Which alternative has the lower equivalent cost.

Soln: Canal.

Annual Maintenance	400
Interest on investment $= 75k(0.06)$	4500
Equivalent Annual cost	4900/-

Culvert

Annual Maintenance	700
Capital recovery.	2906
40k(A/P, 6, 30)	

A company is planning to purchase an advanced m/c centre. 3 orginal manufacturers have responded to its tender whose particulars are tabulated.

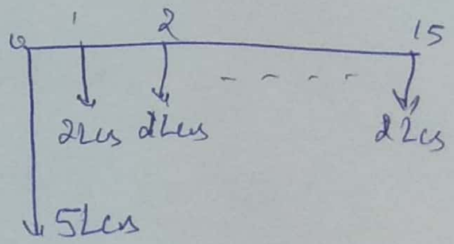
M	Down payment	Yearly equal installement	No. of installment
1	5 Lcs	2 Lcs	15
2	4 Lcs	3 Lcs	15
3	6 Lcs	1.5 Lcs	15

Determine the best alternative based on the annual equivalent method by assuming $i = 20\%$ compounded annualy.

Soln:

M1 → Down Payment, $P = 5$ Lcs, $A = 2$ Lcs $n = 15, i = 20\%$

The annual equivalent cost expression of the above CFD is



$AE_{M_1}(20\%) = 5\text{Lcs}(A/P, 20\%, 15) + 2\text{Lcs}$

$= 5\text{Lcs}(0.2139) + 2\text{Lcs}$

$= 3,06,950.$

M2: $P = 4$ Lcs, $A = 3$ Lcs, $n = 15$, $i = 20\%$.

$AE_{M_2}(20\%) = 4\text{Lcs}(A/P, 20\%, 15) + 3\text{Lcs}$

$= 4\text{Lcs}(0.2139) + 3\text{Lcs}$

3,85,560

M3:

$P = 6$ Lcs, $A = 1.5$ Lcs, $n = 15$, $i = 20\%$

$AE_{M3}(20\%) = 6$ Lcs $(A/P, 20\%, 15) + 1.5$ Lcs

$= 6$ Lcs $(0.21389) + 1.5$ Lcs.

$= 278340$

∴ Company should by from Manufacturer 3.

A company invests in one of the 2 mutually exclusive alternatives. The life of both alternatives is estimated to be 5yrs with the following investments, annual returns & salvage values.

	A	B
Investment (Rs)	-1.5 Lcs	-1.75 Lcs
Annual equal return	60k	70k
Salvage value	15k	~~30~~ 35k.

Determine the best alternative based on the AE method by assuming i = 25%

Soln: Alternative A:

$P = 1.5 Lcs, \quad A = 60k \quad S = 15k, \quad n = 5, \quad i = 25\%$

$AE_A(25\%) = -1.5Lcs(A/P, 25\%, 05) + 60k + 15k(A/F, 25\%, 5)$

$= -1.5Lcs(0.3718) + 60k + 15k(0.1218) = \underline{6057/-}$

Alternative B:

$P = 1.75 Lcs \quad A = 70k, \quad S = 35k, \quad i = 25\%, \quad n = 5$

$AE_B = -1.75Lcs(A/P, 25\%, 5) + 70k + 35k(A/F, 25\%, 5)$

$= -1.75Lcs(0.3718) + 70k + 15k(0.1218) = \underline{9198/-}$

∴ Annual equivalent net return of 'B' is more that A.

∴ Alternative B should be selected.

A company must decide whether to by m/c A (or) m/c B.

	M/c A	M/c B
Initial cost	3Lcs	6 Lcs
Useful years	4	4
Slavage value at the end of m/c life	2Lcs	3Lcs
Annual mainten	30k	—

At 15% interest rate, which m/c should be purchased.

- M/C A

$AE(15\%) = (P-S)\ (A/P, 15, 4) + Si + A$

$= 1Lcs\ (A/P, 15, 4) + (2Lcs \times 0.15) + 30k$

$= 95,030$

Also $= 3Lcs\ (A/P, 15\%, 4) + 30k - 2Lcs\ (A/F, 15\%, 4)$

$= 3Lcs\ (0.3503) + 30k - 2Lcs\ (0.2003)$

$= \underline{95,030/-}$

- M/C B:

$AE(15\%) = (P-S)\ (A/P, i, n) + Si + A$

$= 3Lcs\ (A/P, 15, 4) + (3Lcs \times 0.15) + 0$

$= \underline{1,50,090/-}$

Rate of Return

The rate of return is a percentage that indicates the relative yield on different uses of Capital.

Minimum acceptable rate of return [MARR].

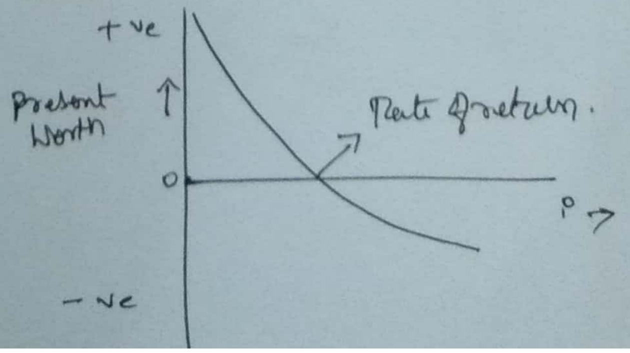
It is the rate set by an organization to designate the lowest level of return that makes an investment acceptable.

IRR:

The internal rate of return is the rate on the unrecovered balance of the investment in a situation where the terminal balance is zero.

ERR:

The external rate of return is the rate of return that is possible to obtain for an investment under current economic conditions.

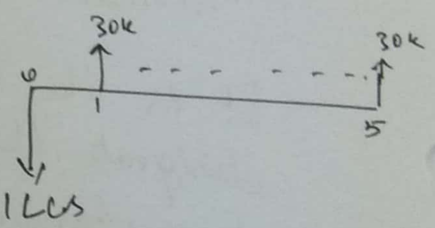


Rate of Return:

A person is planning a new business. The initial outlay & cash flow pattern for the new business are as listed below. The expected life of the business is 5 years. Find the rate of return for the new business.

Period	0	1	2	3	4	5
Cash flow	−1Lcs	30k	30k	30k	30k	30k.

Soln:



PW(i) = −1Lcs + 30k(P/A, i, 5)

when i = 10%

PW(10%) = −1Lcs + 30k(P/A, 10, 5) = 13,724

i = 15%

PW(15%) = −1Lcs + 30k (P/A, 15, 5) = 566

i = 18%

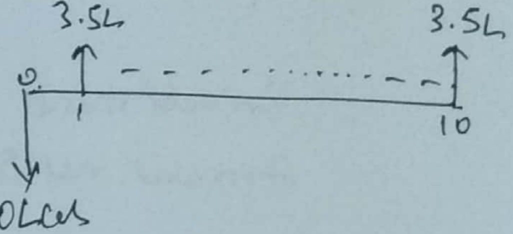
PW(18) = −1Lcs + 30k (P/A, 18, 5) = −6184

$$\therefore \; i = 15\% + \left[\frac{566 - 0}{566 - (-6184)} \times (3\%) \right]$$

~~Generally~~ ∴ i = 15.252%

i⇒

A company is trying to diversify its business in a new product line. The life of the project is 10 years with no salvage value at the end of its life. The initial outlay of the project is 20,00,000. The annual net profit is 3.5Lacs. Find the rate of return for new business



Soln:

$$PW(10\%) = -20Lacs + 3.5L(P/A, 10\%, 10)$$

$$= 150,610$$

$$PW(12\%) = -20L + 3.5L(P/A, 12\%, 10) = -22,430.$$

$$\therefore i = 10\% + \left[\frac{150610 - 0}{150610 - (-22,430)} \times 2\%\right] = 11.74\%.$$

Consider the following CF of a project. Find the rate of return of the project.

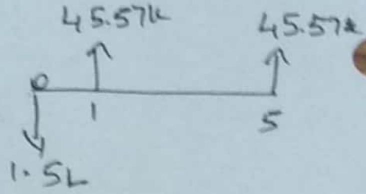
Year	0	1	2	3	4	5
CF	-10k	4k	4.5k	5k	5.5k	6k

A firm has identified 3 mutually exclusive investment proposals whose details are given below. The life of all the 3 alternatives is estimated to be 5 years with negligible salvage value. The min attractive rate of return for the firm is 12%. Find the best alternative based on the rate of return method of comparison.

	Alternative A1	A2	A3
Investment	1.5L	2.1L	2.55L
Annual net income	45570	58260	69K

Soln:

Alternative A1: ✓



$PW(12\%) = -1.5L + 45.57k(P/A, 12, 5) = 14,270.74$

$PW(15\%) = -1.5L + 45.57k(P/A, 15, 5) = 2759.75$

$PW(18\%) = -1.5L + 45570(P/A, 18, 5) = -7493.5$

$\therefore\ P = 15 + \left[\frac{2759.75 - 0}{2759.75 - (-7493.5)}(3\%)\right] = 15.81\%$

A2: $PW(12\%) = -2.1L + 58260(P/A, 12, 5) = 15.65$

$PW(13\%) = -2.1L + 58260(P/A, 13, 5) = -5087.93$

$\therefore\ i = 12\% + \frac{15.65 - 0}{15.65 - (-5087.93)} \times 1\% = 12\%$

A3: $PW(12\%) = -2.55L + 69k(P/A, 12, 5) = -6268.80$

$PW(11\%) = -2.55L + 69k(P/A, 12, 5) = 17.1$

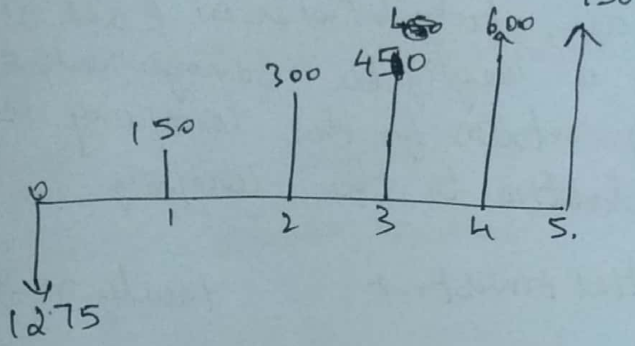
$\therefore\ i = 11 + \frac{17.1 - 0}{17.1 - (-6268.8)} \times 1\%$

$= 11\%$

i for A3 is less than 12%. Ror Interest of A1 is more than A2. ∴ A1 is selected.

Modul

For the CF shown compute the rate of return Amount in Rs.



Soln:

$A_1 = 150, \quad G = 150.$

$\therefore \; A = A_1 + G\,(A/G, i, n) = 150 + 150(A/G, i, 5)$

$PW(10\%) = -1275 + \{[150 + 150(A/G, 10, 5)] \times [P/A, 10, 5]\}$

$= 322.88$

$PW(12\%) = -1275 + \{[150 + 150(A/G, 12, 5)][P/A, 12, 5]\}$

$= 225.28$

$PW(15\%) = -1275 + \{[150 + 150(A/G, 15, 5)][P/A, 15, 5]\}$

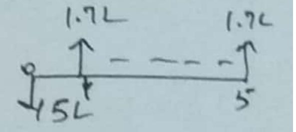
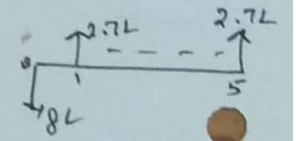
$= 94.11$

$PW(18\%) = -1275 + \{[150 + 150(A/G, 18, 5)][P/A, 18, 5]\}$

$= -21.24.$

$\therefore \; i = 15 + \dfrac{94.11 - 0}{94.11 - (-21.24)} \times 3\% = 17.45\%$

A company is planning to expand its present business activity. It has two alternatives for the expansion programme & the corresponding cash flow are tabulated below. Each alternative has a life of 5 years & a negligible salvage value. The min attractive rate of return for the company is 12%. Suggest the best alternative to the company.

	Initial investment	Yearly revenue	CFD
Alternative 1	5L	1.7L	
Alternative 2	8L	2.7L	

Soln:

$PW_{A_1}(15\%) = -5L + 1.7L(P/A, 15, 5) = 69874$

$PW_{A_1}(17\%) = -5L + 1.7L(P/A, 17, 5) = 43,881$

$PW_{A_1}(20\%) = -5L + 1.7L(P/A, 20, 5) = 8402$

$PW_{A_1}(22\%) = -5L + 1.7L(P/A, 22, 5) = -13188.$

$$\therefore \quad i = 20\% + \frac{8402 - 0}{8402 - (-13188)} \times 2\% = 20.78\%.$$

$PW_{A_2}(15\%) = -8L + 2.7L(P/A, 15, 5) = 105094$

$PW_{A_2}(17\%) = -8L + 2.7L(P/A, 17, 5) = 63811$

$PW_{A_2}(20\%) = -8L + 2.7L(P/A, 20, 5) = 7462$

$PW_{A_2}(22\%) = -8L + 2.7L(P/A, 22, 5) = -26828$

$$\therefore \quad i = 20\% + \frac{7462 - 0}{7462 - (-26828)} \times 2\%$$

$$= 20.435\%.$$

Since RR of A_1 is more than A_2, hence A_1 RR is selected.

Module 4: Depreciation.

Methods of accounting depreciation are as follows.

1) Straight line method of depreciation
2) Declining balance method
3) Sum of years
4) Sinking fund method.
5) Service o/p method.

Straight line method:

In this method of depreciation, a fixed sum is charged as the depreciation amount throughout the life time of an asset such that the accumulated sum at the end of the life of the asset is exactly equal to the purchase value of the asset.

$P \rightarrow$ First cost of the asset. $F \rightarrow$ Salvage value of the asset

$n \rightarrow$ life of the asset $B_t \rightarrow$ Book value of the asset at the end of the period.

$D_t \rightarrow$ Depreciation amount for the period t.

$$D_t = \frac{(P-F)}{n}$$

$$B_t = B_{t-1} - D_t = P - t\left[\frac{P-F}{n}\right]$$

A company has purchased an equipment whose first cost is 1 Lcs with an estimated life of 8 years. The estimated salvage value of the equipment at the end of its lifetime is 20k. Determine the depreciation charge & book value at the end of various years using the straight line method of depreciation.

Soln:

$P = 1$ Lcs, $F = 20k$, $n = 8$

$$D_t = \frac{P-F}{n} = \frac{1,00,000 - 20,000}{8} = 10,000/-$$

End of Year	0	1	2	3	4	5	6	7	8
D_t	10k	10k	10k	10k	10k	10k	10k	10k	10k
B_t [i.e $B_{t-1} - D_t$]	1L	90k	80k	70k	60k	50k	40k	30k	20k

Same problem with $k = 0.2$.

Soln: $P = 1$ Lcs, $F = 20k$, $n = 8$, $k = 0.2$.

$$D_t = k \times B_{t-1}, \quad B_t = B_{t-1} - D_t.$$

End of Years	0	1	2	3	4	5	6	7	8
Depreciation	–	20k	16k	12.8k	10.24k	8.192k	6.553	5242	4194.3
B_t	1Lcs	80k	64k	51.2k	40.96k	32.768	26.214	20.97k	16777.22

Same problem with $k = 0.2$ & ~~n=5~~. Find Book value for period 5 using the declining balance method of depreciation.

Soln:

$$D_t = k(1-k)^{t-1} \times P$$

$$= 0.2(1-0.2)^4 \times 1 \text{ Lcs} = 8192.$$

$$B_t = (1-k)^t P$$

$$= (1-0.2)^5 \times 1,00,000$$

$$= 32,768.$$